Aerospace Technology AT76.04
Orbital Mechanics

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Why Do Satellites ...?

Orbit is the path of satellite around a body (e.g the earth) which is stable under the influence of gravity

1. **Why Do Satellites** Go Around the Earth and Stay in Orbit?
   - The key to getting a spacecraft into orbit is to give it enough horizontal velocity so that the Earth's gravity can't pull it down to the ground the way it pulls down everything else
   - The greater the velocity of a thrown object, the further it will travel
   - The velocity required: more than 7.9 kilometers a second

2. **Why do Satellites** Don't Fall Straight Down to the Earth ?
   - A satellite doesn't fall straight down to the Earth because of its velocity.
   - A satellites orbit there is a perfect balance between the gravitational force or force of gravity due to the Earth, and the centripetal force necessary to maintain the orbit of the satellite.

Satellite Astronomical Velocities

The 1st Astronomical Velocity:

The velocity that a satellite needs to keep it flying close to the surface of the Earth, 7.9 kilometers per second, or the speed at which the Earth's gravity and the centrifugal force of the satellite's rotation are in balance, is known as "the first astronautical velocity."

The 2nd Astronomical Velocity:

If the first space velocity is exceeded, the satellite's orbit becomes an ellipse, and at close to double this velocity, at 11.2 kilometers per second, a speed known as "the second astronautical velocity" or "escape velocity," the satellite escapes the influence of Earth's gravity and leaves its orbit.

Fundamental Physics of Orbital Mechanics

In order to understand how satellite orbits works, we need to know something on about the orbital mechanics which based on some very fundamental physics and some geometry:

1. **Newton's Three Law of Motion and Gravitation**
   - The basis for Classical mechanics.

2. **Geometry of an Ellipse – Elliptical Orbit**
   - Not all orbits are circular - planetary orbits around the Sun. The first satellites had a very eccentric orbit.

3. **Kepler's Three Laws for Orbits**
   - These laws are described for elliptical orbit, and can be used for circular orbit, which is a special case of elliptical orbit.

Fundamental Physics of Orbital Mechanics

1.1 Newton's Three Law of Motion:

(1) Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it (Law of Inertia).

(2) The relationship between an object's mass m, its acceleration a, and the applied force F is $F = ma$.

(3) For every action there is an equal and opposite reaction.

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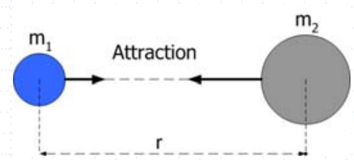
1.2 Newton's Laws of Gravitation - (Law of Universal Gravitation):

The force of gravity or gravitational force $F_{gravity}$ of attraction between any two masses m_1 and m_2 separated by a distance r is:

$$F_{gravity} = \frac{Gm_1 m_2}{r^2} \quad \text{where} \quad G = 6.67 \times 10^{-11}\ N\ m^2/kg^2$$

G is Gravitational constant



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2. Geometry of an Ellipse

F_1 and F_2 : Foci of the ellipse
AC = a: semi-major axis, AB = 2a: major axis
EC = b: semi-minor axis, ED = 2b: minor axis
FC = CF' = c, F F' = 2c

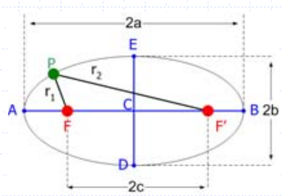
1. For any given point P on the orbit:
$r_1 + r_2 = 2a$ = const

2. Equation of the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b > 0$$

3. $a^2 - b^2 = c^2$

4. Area of Ellipse: $S = \pi ab$



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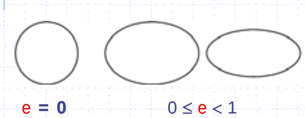
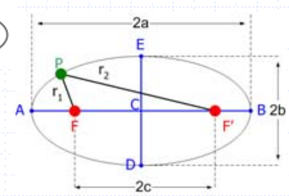
2. Geometry of an Ellipse - Eccentricity

The shape of the ellipse is determined by the eccentricity e

$e = c/a$ and $0 \le e < 1$,

For a circle $e = 0$, since $c = 0$ & F = F'





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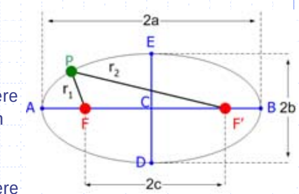
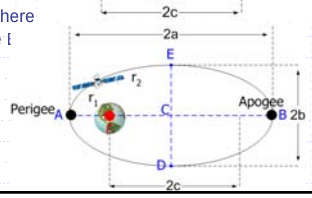
2. Geometry of an Ellipse – Elliptical Orbits

Two special points in the orbit (orbit around the Earth):

1. Perigee: point on the orbit where the satellite is closest to the Earth
perigee = a - c = a(1-e)

2. Apogee: point on the orbit where the satellite is furthest from the E
Apogee = a + c = a(1+e)





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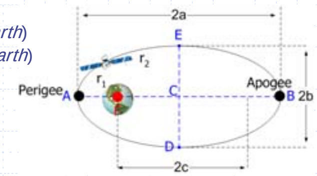
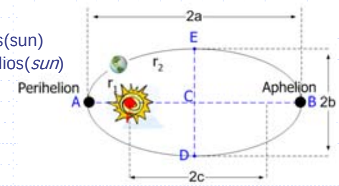
2. Geometry of an Ellipse – Elliptical Orbits

apogee = apon(*away*) + gi(*earth*)
perigee = peri(*toward*) + gi(*earth*)



aphelion = apon(*away*) +ilios(sun)
perihelion = peri(*toward*) + ilios(*sun*)



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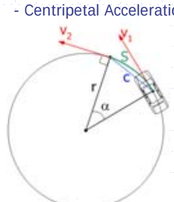
2. Circular orbit

Gravity supplies the necessary centripetal force to hold a satellite in orbit about the earth.

- Centripetal Acceleration a and Centripetal Force F_c

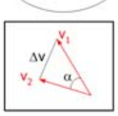


$$\alpha = \frac{S}{r}, \quad \alpha = \frac{\Delta v}{v} \quad \Rightarrow \quad \frac{S}{r} = \frac{\Delta v}{v}$$

$$S = v.\Delta t$$

$$\frac{v.\Delta t}{r} = \frac{\Delta v}{v} \Rightarrow \frac{v.v}{r} = \frac{\Delta v}{\Delta t} = a$$

$$\frac{v^2}{r} = a_{centripetal}$$



Since F = ma, $F_{centripetal} = ma_{centripetal}$

$$F_{centripetal} = m\,\frac{v^2}{r}$$

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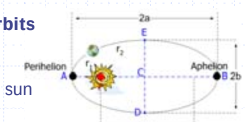
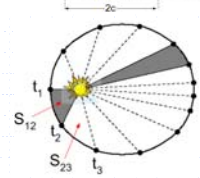
3. Kepler's Three Laws for Orbits

1. The Law of Orbits: All planets move in elliptical orbits, with the sun at one focus.



2. The Law of Areas: A line that connects a planet to the sun sweeps out equal areas in equal time intervals.



3. The Law of Periods: The square of the period of any planet is proportional to the cube of the semi-major axis of its orbit.

$$T^2 = \frac{4\pi^2}{GM} a^3$$ or T^2/a^3 = const

Fundamental Physics of Orbital Mechanics

The orbit can be expressed in terms of the acceleration of gravity at the orbit.

5. **Centripetal Acceleration and Centripetal Force vs. Gravitational Force**

Any motion in a curved path represents accelerated motion, and requires a force directed toward the center of curvature of the path.

This force is called the centripetal force which means "center seeking" force.

Since $F = ma$, $F_{centripetal} = ma_{centripetal}$

$$F_{centripetal} = m\frac{v^2}{r}$$

Fundamental Physics of Orbital Mechanics

6. **Gravitational Force and Weight**

1. All of the objects are attracted to the Earth.
2. The attractive force exerted by the Earth on an objects is the gravitational force
3. The gravitational force is direct to the center of the Earth and its magnitude is call the weight

A freely falling object of mass m experiences an acceleration g acting toward to the center of the earth and its magnitude can be find using the 2nd Law of Newton:

$F = ma$ to a freely falling object of mass m with $a = g$

Therefore: $F_g = mg$ or $P = mg$

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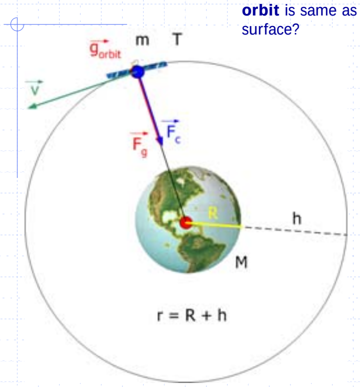
Does the acceleration of gravity **g** at the **orbit** is same as the one on the **Earth** surface?



1. $g_{orbit} \neq g_{earth}$?
2. $g_{orbit} = g_{earth}$?

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Examples 1: Find the gravitational acceleration at the Aqua satellite

The weight W of an object is given by W = mg (Equation 1). The force of gravity, which comes from the law of gravity at the surface of the Earth in the inverse square law (Equation 2) form:

$$F_{gravity} = G\frac{m_1 m_2}{r^2} \quad \text{(Equation 1)}$$

$$W = m_1 g_{orbit} \quad \text{(Equation 2)}$$

Then, $F_{gravity} = W$

$$G\frac{m_1 m_2}{r^2} = m_1 g_{orbit}$$

Replace m_2 by M, then

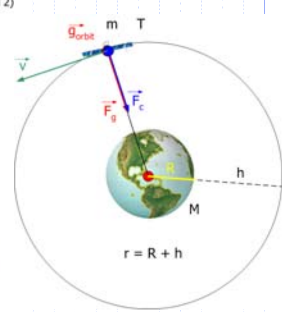
$$\frac{GM}{r^2} = g_{orbit}$$

Here, $r = R + h$, where

$R = 6{,}380$ km (radius of the Earth(average))
$h = 705$ km $= 0.705 \times 10^6$ m (altitude of Aqua satellite)

$M = 5.98 \times 10^{24}$ kg (mass of the Earth)
$G = 6.67 \times 10^{-11}$ Nm^2/kg^2

then $g_{orbit} = 7.956$ m/s^2 ($g_{ground} = 9.81$ m/s^2)



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Examples 2: Find velocity of Aqua satellite at the orbit.

A satellite remains in orbit because the Force of Gravity $F_{gravity}$ or F_g provides Centripetal Force F_c necessary for it to move in a circular orbit. Of course, orbits may be in ellipses, but we restrict our attention to satellite in circular orbit. This keep the radius r and the velocity v = const.

Since F_g provides F_c, then $F_g = F_c$

$$F_{gravity} = G\frac{m_1 m_2}{r^2} \quad \text{(Equation 1)}$$

$$F_c = \frac{m_1 v^2}{r} \quad \text{(Equation 2)}$$

Then, $F_{gravity} = F_c$

$$G\frac{m_1 m_2}{r^2} = \frac{m_1 v^2}{r}$$

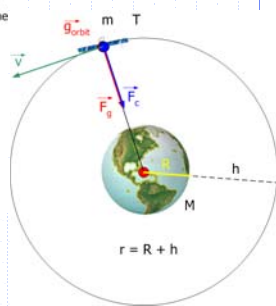
$$v^2_{orbit} = \frac{Gm_2}{r} = \frac{GM}{r} \Rightarrow v_{orbit} = \sqrt{\frac{GM}{r}}$$

Here, r = R + h, where

R = 6,380 km (radius of the Earth(average))
h = 705 km = 0.705×10^6 m (altitude of Aqua satellite)

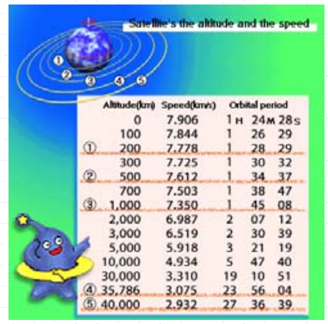
M = 5.98×10^{24} kg (mass of the Earth)
G = 6.67×10^{-11} Nm^2/kg^2

then v_{orbit} = 7.5 km/s



Fundamental Physics of Orbital Mechanics

The Altitude vs. Velocity of Satellites



	Altitude(km)	Speed(km/s)	Orbital period		
	0	7.906	1 H	24 M	28 S
	100	7.844	1	26	29
①	200	7.778	1	28	29
	300	7.725	1	30	32
②	500	7.612	1	34	37
	700	7.503	1	38	47
③	1,000	7.350	1	45	08
	2,000	6.987	2	07	12
	3,000	6.519	2	30	39
	5,000	5.918	3	21	19
	10,000	4.934	5	47	40
	30,000	3.310	19	10	51
④	35,786	3.075	23	56	04
⑤	40,000	2.932	27	36	39

$$v_{orbit} = \sqrt{\frac{GM}{r}}$$

Fundamental Physics of Orbital Mechanics

Examples 3: Find period of Aqua satellite at the orbit.

A satellite remains in orbit because the Force of Gravity $F_{gravity}$ or F_g provides Centripetal Force F_c necessary for it to move in a circular orbit. Of course, orbits may be in ellipses, but we restrict our attention to satellite in circular orbit. This keep the radius r and the velocity v = const.

Since F_g provides F_c, then $F_g = F_c$

$$F_{gravity} = G\frac{m_1 m_2}{r^2} \quad \text{(Equation 1)}$$

$$F_c = \frac{m_1 v^2}{r} \quad \text{(Equation 2)}$$

Then, $F_{gravity} = F_c$

$$= \frac{m_1 v^2}{r}$$

$$v^2_{orbit} = \frac{Gm_2}{r}; \text{ but } v^2_{orbit} = \frac{2\pi r^2}{T^2}$$

$$\text{Then, } \frac{GM}{r} = \frac{2\pi r^2}{T^2} \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM}$$

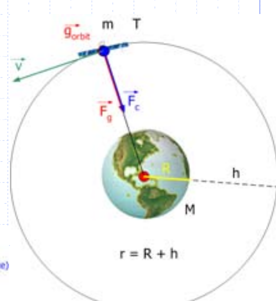
$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Here, r = R + h, where

R = 6,380 km (radius of the Earth(average))
h = 705 km = 0.705×10^6 m (altitude of Aqua satellite)

M = 5.98×10^{24} kg (mass of the Earth)
G = 6.67×10^{-11} Nm^2/kg^2

then T_s = 98.08 min.



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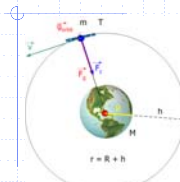
Home Work about Satellite Orbit



Q1. Find the gravitational acceleration at the Aqua satellite at the altitude above the Earth of 705 km.

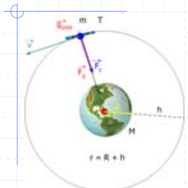
Q2. The International Space Station operates at an altitude of 350km. When final construction is completed, it will have a weight (measured at Earth's surface) of 4.22×10^6 N. What is the weight when in orbit?

Q3. Find the density of the Earth.

Q4. (a) Find the mass of the Sun using the fact that the period of the Earth orbit around the Sun is 365 days and the distance from the Sun is 1.496×10^{11}m
(b) Find the velocity of a satellite on the orbit

Fundamental Physics of Orbital Mechanics

Home Work about Satellite Orbit



Q5. Find the mass of Mars, knowing Mars has two moons: Phobos and Deimos.
T_{Phobos} = 0.32 Days
R_{Phobos} = 9380 km

T_{Deimos} = 1.26 Days
R_{Deimos} = 23,460 km

Q6. How far the Geosynchronous satellite is above the surface of the Earth?

Fundamental Physics of Orbital Mechanics

Lab Sessions with STK

Please implement the following lab sessions with STK and verify the result with manual calculations.

Q1. The **1st scenario** is the near-polar orbit with Apogee Altitude R_a = 12,000 km and Perigee Altitude R_p = 400 km, Longitude of Ascending node = -100 deg.
Observe the velocity of the satellite at Perigee and Apogee
Calculate the velocity of the satellite at these two points

The **2nd scenario** is also the near-polar orbit with Apogee Altitude Ra = 12,000 km and Perigee Altitude Rp = 12,000 km, Longitude of Ascending node = -100 deg.
Observe the velocity of the satellite at Perigee and Apogee.
What we can explain about the velocity of the satellite in the 2 cases? (Scenarios 1st and 2nd). Please compare the velocity of the satellite in the two cases.

(continued on next slide)

Fundamental Physics of Orbital Mechanics

Lab Sessions with STK

Q2. Based on the information given in Q1, assume that the satellite in two cases are identical (weight,…). Please compare the period of the satellite in each orbit and calculate the Period for the satellite in elliptical orbit.

Q3. From the results obtained from previous questions, as R_p is 400km and Ra is 12,000km as shown in STK, please prove that: $R_p + R_a = 2a$ by knowing $R_a = a(1+e)$ and $R_p = a(1-e)$