Orbital Mechanics

Orbit Perturbations

Theory & Practice

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Orbit Perturbations

- Third-body Perturbations
- Perturbations due to non-spherical Earth
  - J_2 Effect
- Perturbation from the Atmospheric Drag
- Perturbation from the Solar Radiation

1. Third-body Perturbations

- For nearly circular orbits the equations for the secular rates of change resulting from the Sun and Moon are :
- Longitude of the ascending node:
- Ω_{Moon} = -0.00338 cos(i)/n
- Ω_{Sun} = -0.00154 cos(i)/n

- Argument of perigee:
- ω_{Moon} = 0.00169(4-5sin^{2}i)/n
- ω_{Sun} = 0.0077(4-5sin^{2}i)/n

1. Third-body Perturbations

- PROBLEM 1
- Calculate the perturbations in longitude of the ascending node and argument of perigee caused by the Moon and Sun for the International Space Station orbiting at an altitude of 400 km, an inclination of 51.6 degrees, and with an orbital period of 92.6 minutes.

- SOLUTION, Given: i = 51.6 degrees
- n = 1436 / 92.6 = 15.5 revolutions/day

- Ω_moon = -0.00338 x cos(i) / n
- Ω_moon = -0.00338 x cos(51.6) / 15.5
- Ω_moon = -0.000135 deg/day

- ω_sun = -0.00154 x cos(i) / n
- ω_sun = -0.00154 x cos(51.6) / 15.5
- ω_sun = -0.0000617 deg/day

1. Third-body Perturbations

- PROBLEM 1 (Continued)
- Calculate the perturbations in longitude of the ascending node and argument of perigee caused by the Moon and Sun for the International Space Station orbiting at an altitude of 400 km, an inclination of 51.6 degrees, and with an orbital period of 92.6 minutes.

- SOLUTION, Given: i = 51.6 degrees
- n = 1436 / 92.6 = 15.5 revolutions/day

- ω_moon = 0.00169 x (4 - 5 x sin2 i) / n
- ω _moon = 0.00169 x (4 - 5 x sin2 51.6) / 15.5
- ω_moon = 0.000101 deg/day

- ω_sun = 0.00077 x (4 - 5 x sin2 i) / n
- ω_sun = 0.00077 x (4 - 5 x sin2 51.6) / 15.5
- ω_sun = 0.000046 deg/day

2. Perturbations due to Non-spherical Earth

- When developing the two-body equations of motion, we assumed the Earth was a spherically symmetrical, homogeneous mass. In fact, the Earth is neither homogeneous nor spherical.

- The most dominant features are a bulge at the equator, a slight pear shape, and flattening at the poles. For a potential function of the Earth, we can find a satellite's acceleration by taking the gradient of the potential function. The most widely used form of the geo-potential function depends on latitude and geo-potential coefficients, ***Jn***, called the *zonal coefficients*.

- The potential generated by the non-spherical Earth causes periodic variations in all the orbital elements. The dominant effects, however, are secular variations in longitude of the ascending node and argument of perigee because of the Earth's oblateness, represented by the J2 term in the geo-potential expansion. The rates of change of Ω and ω due to J2 are shown in the next slide:

2. Perturbations due to Non-spherical Earth

- $\Omega_{J2} = -1.5nJ_2(R_E/a)^2(\cos i)(1-e^2)^{-2}$
- $\approx -2.06474 \times 10^{14}\ a^{-7/2}\ (\cos i)(1-e^2)^{-2}$

- $\omega_{J2} = 0.75nJ_2(R_E/a)^2(4-5\sin^2 i)\ (1-e^2)^{-2}$
- $\approx 1.03237\text{x}10^{14}\ a^{-7/2}\ (4-5\sin^2 i)\ (1-e^2)^{-2}$

where
- n is the mean motion in degrees/day,
- J_2 has the value 0.00108263,
- R_E is the Earth's equatorial radius,
- a is the semi-major axis in kilometers,
- i is the inclination,
- e is the eccentricity, and Ω and ω are in degrees/day.

- For satellites in GEO and below, the J2 perturbations dominate;
- for satellites above GEO the Sun and Moon perturbations dominate.

2. Perturbations due to Non-spherical Earth

- PROBLEM 2

A satellite is in an orbit with a semi-major axis of 7,500 km, an inclination of 28.5 degrees, and an eccentricity of 0.1. Calculate the J2 perturbations in longitude of the ascending node and argument of perigee.

Given: a = 7,500 km i = 28.5 degrees e = 0.1

$\Omega\_J_2 = -2.06474\text{x}10^{14} \text{ x } a^{-7/2} \text{ x } (\cos i) \text{ x } (1 - e^2)^{-2}$

$\Omega\_J_2 = -2.06474\text{x}10^{14} \text{ x } (7{,}500)^{-7/2} \text{ x } (\cos 28.5) \text{ x } (1 - (0.1)^2)^{-2}$

$\Omega\_J_2 = -5.067$ deg/day

$\omega\_J_2 = 1.03237\text{x}10^{14} \text{ x } a^{-7/2} \text{ x } (4 - 5 \text{ x } \sin^2 i) \text{ x } (1 - e2)^{-2}$

$\omega\_J_2 = 1.03237\text{x}10^{14} \text{ x } (7{,}500)^{-7/2} \text{ x } (4 - 5 \text{ x } \sin^2 28.5) \text{ x } (1 - (0.1)^2)^{-2}$

$\omega\_J_2 = 8.250$ deg/day

3. Perturbations from Atmospheric Drag

- The drag force F_D on a body acts in the opposite direction of the velocity vector and is given by the equation:

$F_D = (1/2)\ C_D\ \rho V^2 A$

where
CD is the drag coefficient,
ρ is the air density,
V is the body's velocity, and
A is the area of the body normal to the flow.

The drag coefficient is dependent on the geometric form of the body and is generally determined by experiment. Earth orbiting satellites typically have very high drag coefficients in the range of about 2 to 4. Air density is given by the Atmosphere Properties Table (see next slide).

Atmospheric Scale Height & Density, to 35,786 km

Altitude (km)	Scale Height (km)	Atmospheric Density	
		Mean (kg/m^3)	Maximum (kg/m^3)
0	8.4	1.225	1.225
100	5.9	5.25E-7	5.75E-7
150	25.5	1.73E-9	1.99E-9
200	37.5	2.41E-10	3.65E-10
250	44.8	5.97E-11	1.20E-10
300	50.3	1.87E-11	4.84E-11
350	54.8	6.66E-12	2.18E-11
400	58.2	2.62E-12	1.05E-11
450	61.3	1.09E-12	5.35E-12
500	64.5	4.76E-13	2.82E-12
550	68.7	2.14E-13	1.53E-12
600	74.8	9.89E-14	8.46E-13
650	84.4	4.73E-14	4.77E-13
700	99.3	2.36E-14	2.73E-13
750	121	1.24E-14	1.59E-13
800	151	6.95E-15	9.41E-14
850	188	4.22E-15	5.67E-14
900	226	2.78E-15	3.49E-14
950	263	1.98E-15	2.21E-14
1,000	296	1.49E-15	1.43E-14
1,250	408	5.70E-16	2.82E-15
1,500	516	2.79E-16	1.16E-15
2,000	829	9.09E-17	3.80E-16
2,500	1220	4.23E-17	1.54E-16
3,000	1590	2.54E-17	7.09E-17
3,500	1900	1.77E-17	3.67E-17
4,000	2180	1.34E-17	2.11E-17
4,500	2430	1.06E-17	1.34E-17
5,000	2690	8.62E-18	9.30E-18
6,000	3200	6.09E-18	5.41E-18
7,000	3750	4.56E-18	3.74E-18
8,000	4340	3.56E-18	2.87E-18
9,000	4970	2.87E-18	2.34E-18
10,000	5630	2.37E-18	1.98E-18
15,000	9600	1.21E-18	1.16E-18
20,000	14600	7.92E-19	8.42E-19
25,000	20700	5.95E-19	6.81E-19
30,000	27800	4.83E-19	5.84E-19
35,000	36000	4.13E-19	5.21E-19
35,786	37300	4.04E-19	5.12E-19

3. Perturbations from Atmospheric Drag

- For circular orbits we can approximate the changes in semi-major axis, period, and velocity per revolution using the following equations:

- $\Delta a_{rev} = (-2\pi C_D A \rho a^2)/m$
- $\Delta P_{rev} = (-6\pi^2 C_D A \rho a^2)/m$
- $\Delta V_{rev} = (\pi C_D A \rho a V)/m$

where
- *a* is the semi-major axis,
- *P* is the orbit period, and
- V, A and *m* are the satellite's velocity, area, and mass respectively.

3. Perturbations from Atmospheric Drag

- A rough estimate of a satellite's lifetime, *L*, due to drag can be computed from
- $L = -H/\Delta a_{rev}$
- where *H* is the atmospheric density scale height

3. Perturbations from Atmospheric Drag

- PROBLEM 3

A satellite is in a circular Earth orbit at an altitude of 400 km. The satellite has a cylindrical shape 2 m in diameter by 4 m long and has a mass of 1,000 kg. The satellite is traveling with its long axis perpendicular to the velocity vector and it's drag coefficient is 2.67. Calculate the perturbations due to atmospheric drag and estimate the satellite's lifetime.

3. Perturbations from Atmospheric Drag

SOLUTION:

Given:

- a = (6,378.14 + 400) x 1,000 = 6,778,140 m
- A = 2 x 4 = 8 m^2
- m = 1,000 kg
- C_D = 2.67

From Atmosphere Properties:

ρ = 2.62x10-12 kg/m^3 and H = 58.2 km

3. Perturbations from Atmospheric Drag

V = SQRT[GM / a]

V = SQRT[3.986005x1014 / 6,778,140]

V = 7,669 m/s

Then apply the equations below, to calculate for the perturbations due to atmospheric drag for semi-major axis, period, and velocity per revolution:

- $\Delta a_{rev} = (-2\pi C_D A \rho a^2)/m$
- $\Delta P_{rev} = (-6\pi^2 C_D A \rho a^2)/m$
- $\Delta V_{rev} = (\pi C_D A \rho a V)/m$

e.g.: $\Delta a_{rev} = (-2 \times \pi \times C_D \times A \times \rho \times a^2) / m =$

$= (-2 \times \pi \times 2.67 \times 8 \times 2. \times 2.62\text{x}10^{-12} \times 6{,}778{,}140^2)/1{,}000 = -16.2$ m

3. Perturbations from Atmospheric Drag

- Estimate the satellite's lifetime
- L ~ -H / Δa_{rev}
- L ~ -(58.2 x 1,000) / -16.2
- L ~ 3,600 revolutions

4. Perturbations from Solar Radiation

- $a_{Solar-Rad} \sim -4.5 \times 10^{-6} (A_{sat}/m_{sat})$
- in which unit is [m/s^2]